Eur. Phys. J. D **38**, 397–404 (2006)
DOI: 10.1140/epjd/e2006-00017-3

THE EUROPEAN PHYSICAL JOURNAL D

Manipulating the phase of a single-mode micromaser by polarized three-level atoms

W.-H. Sun[1], S.-D. Du[1,a], and X.-S. Chen[2]

[1] Department of Physics, Fudan University, Shanghai 200433, P.R. China
[2] National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, Chinese Academy of Sciences, 500 Yu-Tian Road, Shanghai 200083, P.R. China

Received 1st August 2005 / Received in final form 26 November 2005
Published online 24 January 2006 –

Abstract. We investigate the phase probability distribution (PPD) of a single-mode micromaser pumped by atoms injected in the most general case, i.e. in the superposition of the upper, intermediate and lower states by the Monte Carlo wave function approach. The phase properties of the cavity mode are greatly influenced by the relative phases and the amplitudes of the polarized atoms, and the detunings between the atom and cavity. The cavity field has a single preferred phase if the cavity is pumped by the atoms in the superposition of the upper and intermediate states or of the intermediate and lower states. However, a double-peak feature appears in the PPD of the cavity field when the cavity is pumped by the atoms in the superposition of the upper and lower states. With appropriate detunings, the double peaks become narrower and more remarkable, which shows the better defined phase of the cavity field, as compared to the resonant case. The PPD displays complicated characteristics when the cavity is pumped by the atoms in the superposition of the upper, intermediate and lower states. The phase distribution changes from a single peak to double peaks and to another single peak when we modulate the phase of the intermediate state, which has been explained in the semi-classical radiation theory.

PACS. 42.50.Pq Cavity quantum electrodynamics; micromasers – 42.50.Ar Photon statistics and coherence theory – 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption

1 Introduction

With the breakthrough in technology, cavity quantum electrodynamics (QED) in the strong coupling regimes has become a very active field of research ranging from experimental tests on fundamental problems in quantum mechanics to the implementation of basic processes for quantum information [1]. The basic core of cavity QED is involved with the so-called Jaynes-Cummings model (JCM) which describes the interaction of a single two-level atom with a single mode of the quantized electromagnetic field [2]. A micromaser is the experimental realization of the JCM or other similar models [3], which mainly includes one- and two-photon transition processes. Many non-classical effects, such as trapping states [4], and Fock states [5], have been observed in the one-photon, two-level micromaser. Very recently, many authors have investigated various one-photon micromasers with initial atomic coherence [6–8]. In parallel, two-photon micromasers are naturally another important subject of research [9–13], since the two-photon process can emit photons in pairs, so that it leads to some interesting phenomena greatly different from the one-photon process. About 20 years ago, the theory of a two-photon micromaser was firstly established by Brune et al. [9] and Davidovich et al. [10]. Then the two-photon micromaser was experimentally realized by Brune et al. [11]. Later, the theory of this two-photon micromaser was further developed by Ashraf et al. [12] and Toor et al. [13]. However, little attention has been devoted to field's properties in the two-photon micromaser with initial atomic coherence, so far.

How to build a coherent field in a micromaser is a very meaningful question in quantum optics. The energy entanglement between the atom and the cavity photons does not imply the excitation of coherences of the cavity field: the steady-state density matrix is always diagonal even if the field is initially prepared in a coherent state. Actually, cavity mode coherences can be induced and exhibited at steady state only if the cavity is pumped by atoms prepared in superposition states. In this case the cavity mode can develop a preferred phase, i.e. an appreciable value of the electromagnetic field expectation value. In this paper, we intend to investigate the field' phase properties of the two-photon mciromaser pumped by atoms in the

[a] e-mail: sddu@fudan.ac.cn

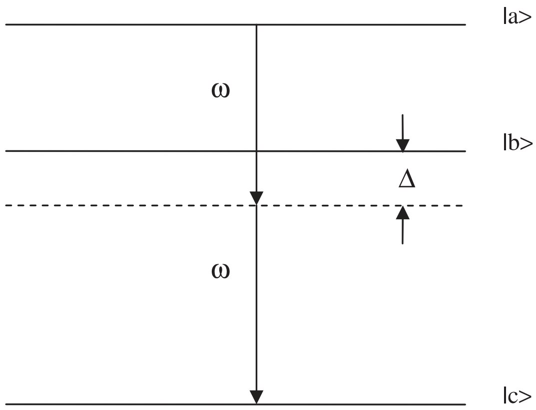

Fig. 1. Atomic level configurations used throughout this paper.

superposition of the upper, intermediate and lower states by the Monte Carlo wave-function (MCWF) [14,15] approach. We focus on how to manipulate the phase of the cavity field by adjusting the relative phases and the amplitudes of the polarized atoms, and the detunings between the atom and cavity.

The present paper is organized as follows. In Section 2 we introduce the theory of the two-photon micromaser with initial atomic coherence, and derive the master equation of the cavity mode. In Section 3 we investigate the phase probability distribution of the cavity mode by the MCWF approach. In Section 4 we discuss an experimental observation of our results and close our paper with some final remarks in Section 5.

2 Model and theory

As we know, a true micromaser consists of a single mode high-Q resonator in which a monoenergetic beam of excited atoms are injected at such a low flux that, at most, one atom at a time is present inside the cavity. In our model, the injected atom is in a cascade three-level configuration as shown in Figure 1. The exact two-photon resonance between the upper state $|a\rangle$ and the lower state $|c\rangle$ is assumed and the intermediate state $|b\rangle$ is detuned from the exact one-photon resonance, with the detuning defined by

$$\Delta = \omega - (\omega_a - \omega_b) = (\omega_b - \omega_c) - \omega, \tag{1}$$

where ω is the frequency of the resonant mode of the micromaser cavity. The frequencies related with the atomic states $|a\rangle$, $|b\rangle$, and $|c\rangle$ are ω_a, ω_b, and ω_c, respectively. The Hamiltonian in the interaction picture can be expressed as:

$$\begin{aligned} H = &\hbar g_1(\hat{a}\sigma_{ab}e^{-i\Delta t} + \hat{a}^+\sigma_{ba}e^{i\Delta t}) \\ &+ \hbar g_2(\hat{a}\sigma_{bc}e^{i\Delta t} + \hat{a}^+\sigma_{cb}e^{-i\Delta t}), \end{aligned} \tag{2}$$

where g_1 and g_2 are the coupling constants for the transitions $|a\rangle \to |b\rangle$ and $|b\rangle \to |c\rangle$, respectively. The quantity $\sigma_{ij} = |i\rangle\langle j|$ is the atomic polarization operator. As a result, the temporal evolution operator U can be theoretically acquired in this interaction picture, which obeys the following equation:

$$i\hbar\frac{\partial U}{\partial t} = HU. \tag{3}$$

Through tedious mathematical calculations, one can obtain the following expression:

$$U(\tau) = \begin{pmatrix} 1 - \hat{a}G_{11}\hat{a}^+ & -ig_1\hat{a}Q & -\hat{a}G_{12}\hat{a} \\ -ig_1Q^+\hat{a}^+ & O^+ & -ig_2Q^+\hat{a} \\ -\hat{a}^+G_{21}\hat{a}^+ & -ig_2\hat{a}^+Q & 1 - \hat{a}^+G_{22}\hat{a} \end{pmatrix}, \tag{4}$$

where

$$\begin{aligned} \Lambda &= g_1^2\hat{a}^+\hat{a} + g_2^2\hat{a}\hat{a}^+ + \Delta^2/4, \\ \Lambda_{11} &= g_1^2\big/\left(g_1^2\hat{a}^+\hat{a} + g_2^2\hat{a}\hat{a}^+\right), \\ \Lambda_{22} &= g_2^2\big/\left(g_1^2\hat{a}^+\hat{a} + g_2^2\hat{a}\hat{a}^+\right), \\ \Lambda_{12} &= \Lambda_{21} = g_1g_2\big/\left(g_1^2\hat{a}^+\hat{a} + g_2^2\hat{a}\hat{a}^+\right), \\ Q &= \sin(\sqrt{\Lambda}\tau)e^{-i\Delta\tau/2}\Big/\sqrt{\Lambda}, \\ O &= (\cos(\sqrt{\Lambda}\tau) + (i\Delta\sin(\sqrt{\Lambda}\tau)\Big/2\sqrt{\Lambda}))e^{-i\Delta\tau/2}, \\ G_{11} &= \Lambda_{11}(1-O),\ G_{12} = G_{21} = \Lambda_{12}(1-O), \\ G_{22} &= \Lambda_{22}(1-O). \end{aligned}$$

Assuming that the atoms injected into the cavity are in a superposition state of the upper state $|a\rangle$, the intermediate state $|b\rangle$ and the lower state $|c\rangle$, which can be written as:

$$|\psi_A\rangle = C_a|a\rangle + C_b|b\rangle + C_c|c\rangle. \tag{5}$$

If such atoms enter the cavity, the initial density matrix of our system is:

$$\rho_{AF}(t_0) = \rho_A(t_0) \otimes \rho_F(t_0) = \begin{pmatrix} \rho_{aa} & \rho_{ab} & \rho_{ac} \\ \rho_{ba} & \rho_{bb} & \rho_{bc} \\ \rho_{ca} & \rho_{cb} & \rho_{cc} \end{pmatrix} \otimes \rho_F(t_0), \tag{6}$$

where $\rho_F(t_0)$ is the initial field density matrix and

$$\rho_{ij} = C_iC_j^* \quad (i,j = a,b,c).$$

By using equation (4) it is easy to write the field reduced density matrix after one atom has passed through the cavity:

$$\begin{aligned} \rho_F(t_0+\tau) &= \mathrm{Tr}_A[U(\tau)\rho_{AF}(t_0)U^+(\tau)] \\ &= \hat{J}\rho_F(t_0)\hat{J}^+ + \hat{K}\rho_F(t_0)\hat{K}^+ + \hat{L}\rho_F(t_0)\hat{L}^+ \\ &= M_g\rho_F(t_0), \end{aligned} \tag{7}$$

where Tr_A represents the trace over the atomic variables and we have defined:

$$\begin{aligned} \hat{J} &= C_a(1 - \hat{a}G_{11}\hat{a}^+) + C_b(-ig_1\hat{a}Q) + C_c(-\hat{a}G_{12}\hat{a}), \\ \hat{K} &= C_a(-ig_1Q^+\hat{a}^+) + C_bO^+ + C_c(-ig_2Q^+\hat{a}), \\ \hat{L} &= C_a(-\hat{a}^+G_{21}\hat{a}^+) + C_b(-ig_2\hat{a}^+Q) + C_c(1 - \hat{a}^+G_{22}\hat{a}). \end{aligned}$$

According to the usual statistical treatment for a Poissonian pumping, one can obtain the following master equation (ME) (here we have ignored the subscript F for convenience):

$$\dot{\rho}(t) = \hat{\ell}\rho(t) + N_{ex}(M_g - 1)\rho(t), \tag{8}$$

where N_{ex} is the average number of atoms that traverse the cavity during the lifetime of the field, and $\hat{\ell}$ is the Liouville super-operator, which can be expressed as:

$$\hat{\ell}\rho(t) = \frac{1+n_b}{2}\left[2\hat{a}\rho(t)\hat{a}^+ - \hat{a}^+\hat{a}\rho(t) - \rho(t)\hat{a}^+\hat{a}\right] + \frac{n_b}{2}\left[2\hat{a}^+\rho(t)\hat{a} - \hat{a}\hat{a}^+\rho(t) - \rho(t)\hat{a}\hat{a}^+\right], \tag{9}$$

with n_b being the average number of thermal photons. Note that in equation (8) atomic coherence induces off-diagonal elements of the cavity mode density matrix. In general, it is rather difficult to obtain a analytical solution to equation (8) in the steady state. Here we apply the MCWF approach to simulate the master equation given by equation (8).

By applying equations (7) and (9) to equation (8), the ME can be written in the Lindblad [16] form as follows:

$$\begin{aligned}\dot{\rho}(t) &= \sum_{K=1}^{5}\left\{\hat{C}_k\rho(t)\hat{C}_k^+ - \frac{1}{2}\left[\hat{C}_k^+\hat{C}_k\rho(t) + \rho(t)\hat{C}_k^+\hat{C}_k\right]\right\},\\ &= \sum_{K=1}^{5}\left\{\hat{C}_k\rho(t)\hat{C}_k^+\right\} - \frac{i}{\hbar}\left[\hat{H}_{\mathit{eff}}\rho(t) + \rho(t)\hat{H}_{\mathit{eff}}^+\right],\end{aligned} \tag{10}$$

where the jump operators

$$\hat{C}_1 = \sqrt{N_{ex}}\hat{J}, \quad \hat{C}_2 = \sqrt{N_{ex}}\hat{K}, \quad \hat{C}_3 = \sqrt{N_{ex}}\hat{L}, \quad \hat{C}_4 = \sqrt{1+n_b}\hat{a}, \quad \hat{C}_5 = \sqrt{n_b}\hat{a}^+, \tag{11}$$

describe the instantaneous changes of the cavity field state induced by coherent ($C_{1,2,3}$) and incoherent ($C_{4,5}$) processes, and $\hat{H}_{\mathit{eff}}$ is an effective non-unitary Hamiltonian

$$\hat{H}_{\mathit{eff}} = -i\frac{\hbar}{2}\left[N_{ex} + n_b + (1+2n_b)\hat{a}^+\hat{a}\right], \tag{12}$$

which describes the continuous evolution of the cavity field between the quantum jumps.

The form (10) of the ME is the most suitable for the application of the MCWF approach [14]. By this approach we can calculate the time evolution and steady-state values of all relevant observables and distributions. Here we are only interested in the phase probability distribution (PPD) of the cavity mode as defined by Pegg and Barnett [17]:

$$P(\theta,\tau) = \frac{1}{2\pi}\sum_{m,n=0}^{\infty}\rho_{mn}^{s}(\tau)\exp\left[i(m-n)\theta\right], \tag{13}$$

where $\rho_{mn}^{s}(\tau)$ is the field's density matrix elements in the steady state.

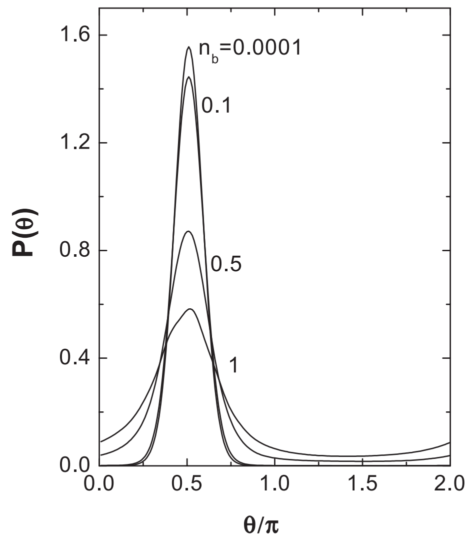

Fig. 2. The PPD of the cavity field influenced by the thermal photons at a fixed interaction time when the cavity is pumped by the atoms injected in the superposition state with $|C_a| = |C_b| = \sqrt{2}/2$, $C_c = 0$, and $\phi = 0$.

3 Numerical results

In this section we investigate the phase properties of the cavity field based on equation (13) by the MCWF approach. On transferring of atomic coherence to the cavity field, we consider the following three factors: atomic polarization amplitude and phase, and the atom-field detuning. For simplicity, we assume that $g_1 = g_2 = g$. In general, the following parameter values are fixed throughout this paper:

$$N_{ex} = 10, \quad \Delta = 0, n_b = 10^{-4}, \quad \text{and} \quad g\tau = 0.13\pi, \tag{14}$$

if they are not noted specially.

3.1 The case when the atom is in the superposition of the upper and intermediate states or of the intermediate and lower states

First consider the case when the atoms are initially prepared in the superposition of the upper and intermediate states: $C_a = \sqrt{2}/2e^{i\phi}$, $C_b = \sqrt{2}/2$, and $C_c = 0\,(\phi = 0)$, where ϕ is the relative phase between the upper and intermediate states. The effect of temperature is presented in Figure 2. As seen from this figure, the single peak of the PPD becomes broader and less remarkable with the increase of the thermal photons. Even n_b is up to 1, the phase of the cavity field can still have a well-defined preferred value. When $n_b \leq 1$, therefore, the pumping atom can efficiently transfer its coherence to the cavity field. On the other hand, one can observe from Figure 2 that a peak occurs at the value of $\theta = 0.5\pi$. This phase, which is determined by the Hamiltonian form of the system, has no any physical meaning. Therefore it should be viewed as a phase reference.

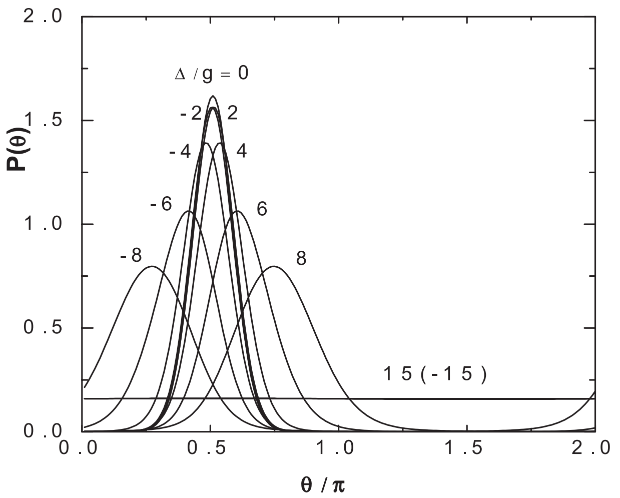

Fig. 3. The PPD of the cavity field for both different positive and negative detunings when the cavity is pumped by the atoms injected in the superposition state with $|C_a| = |C_b| = \sqrt{2}/2$, $C_c = 0$, and $\phi = 0$.

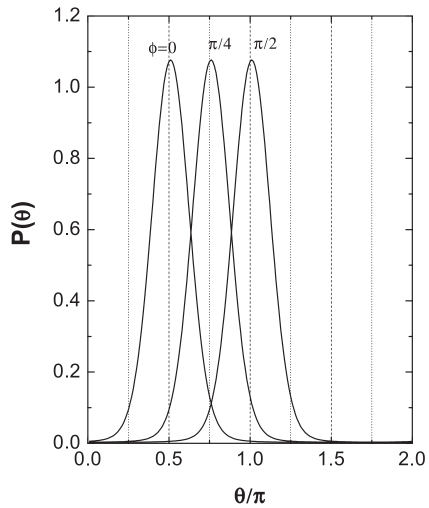

Fig. 4. The PPD of the cavity field pumped by the coherent atoms with different phases. Atoms are in the superposition state with $\phi = 0, \pi/4, \pi/2$ and $C_a = 0$, $|C_b| = |C_c| = \sqrt{2}/2$.

In Figure 3, we show the phase shift with different detunings at a fixed interaction time. For positive detunings, the single peak moves to a larger value on the right side. For negative detunings, however, the single peak moves to the reversal direction, as compared to the case of positive detunings. When we increase the atom-field detuning, the PPD becomes broader and broader, and finally becomes a straight line. In resonance case, the intermediate state provides a temporary stage of jumping from $|a\rangle \to |b\rangle \to |c\rangle$ for the electron, and the best transfer of phase coherence from the atom to the field in the cavity. With increasing the detunings, the role of the intermediate level becomes less and less important, and when $\Delta/g = 15$ (-15), the electron in the upper level will transit to the lower level directly without the help of the intermediate level. Then the preferred phase of the cavity field will becomes disordered at a large detuning. Thus we can modulate the detuning in order to get a preferred phase. These results show the possibility of manipulating the phase of the cavity by adjusting the detuning.

When the cavity is pumped by the atoms in the superposition of the intermediate and lower states, which can be expressed as follows: $C_a = 0$, $C_b = \sqrt{2}/2e^{i\phi}$, and $C_c = \sqrt{2}/2$, the PPD of the cavity field is similar to the corresponding one ($n_b = 0.0001$) in Figure 2, but the peak values are smaller than the latter; see Figure 4. If we adjust the relative phase of the coherent atoms, the single peak will be shifted. In Figure 4, when $\phi = 0$, the peak appears at $\theta = \pi/2$. With the increase of ϕ to $\pi/4$, $\pi/2$, the peak moves to $\theta = 3\pi/4, \pi$. So the peak is located at $\theta = \pi/2 + \phi$ according to the phase ϕ of the intermediate state. It is very obvious that we can control the phase of the cavity field by adjusting the relative phase ϕ of the polarized atom.

The single peak of the phase distribution is considered to be induced by the one-photon transition process. Since the atom transits from $|a\rangle \to |b\rangle$ (if the atoms are injected in the superposition of the upper and intermediate states) or $|b\rangle \to |c\rangle$ (if the atoms are injected in the superposition of the intermediate and lower states), one photon with definite phases is emitted which has been figured out, for example, in the phase distribution of Figure 4. The two-photon transition process cannot contribute to the phase of the cavity field, since two photons with no definite phases are emitted by the atom transiting from $|a\rangle \to |c\rangle$.

It should be also noted that for the resonant interaction it is not surprising that there is little difference between the PPDs of the cavity field pumped by polarized three-level atoms and by polarized two-level atoms [6,7], since the two systems are all determined by one-photon transition process.

3.2 The case when the atom is in the superposition of the upper and lower states

If the cavity is pumped by the atoms in the superposition of the upper and the lower states which can be written as follows: $C_a = \sqrt{2}/2e^{i\phi}$, $C_b = 0$, and $C_c = \sqrt{2}/2$, double peaks appear in the PPD of the cavity field; see Figure 5. The two peaks are all the same, and there is a spacing of π between them. When $\phi = 0$, one peak appears at $\theta = \pi/2$, the other one at $\theta = -\pi/2$. With the increase of ϕ, the double peaks move right to larger θ. From Figure 5, when $\phi = \pi/2\,(-\pi/2)$, the double peaks are shifted to the right (left) side by $\pi/4$. So the double peaks are located at $\pm\pi/2 + \phi/2$, respectively.

Comparing with the single-peak structure of the PPD in Section 3.1, the double-peak one in Figure 5 should be attributed to the occurrence of a pure two-photon transition process. When the atom transits from $|a\rangle \to |b\rangle$ or $|b\rangle \to |c\rangle$, one photon is emitted. However, the phase of this photon is random due to no definite phases in the intermediate state. Thus the one-photon process cannot

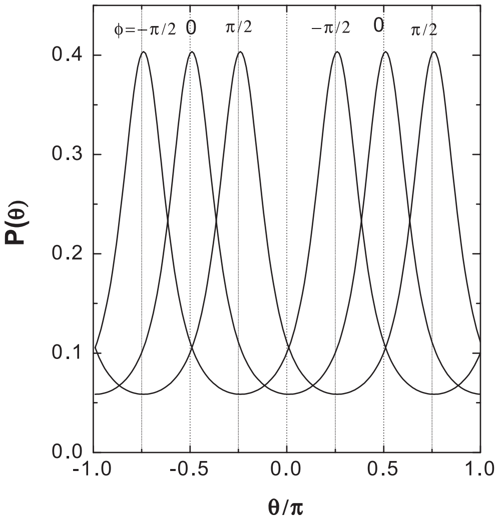

Fig. 5. The PPD of the field in the cavity pumped by the coherent atoms with different phases. Atoms are in superposition states with $C_a = \sqrt{2}/2e^{i\phi}$, $C_b = 0$, $C_c = \sqrt{2}/2$ and $\phi = 0, \pm\pi/2$.

contribute to the double peaks of the phase distribution and the phase of the field is induced only by the pure two-photon process. In our system, we can conclude the occurrence of the two-photon process with appearance of the double peaks of the phase distribution.

Consider a more general case including detunings. Here we choose another set of the related parameters: $|C_a/C_c| = 0.88$ and $g\tau = 0.482\pi$. The PPD versus detunings is shown in Figure 6. The phase distribution still displays a two-peak structure. For the resonant case, the two peaks become lower and broader than the corresponding one in Figure 5. With the increment of the detunings, the two peaks are shifted to the right side; see Figures 6a and 6b. By increasing the detuning from $0 \rightarrow 8g$, the two peaks become narrower and more remarkable; see Figure 6a. If the detuning is further enhanced from $8g \rightarrow 20g$, on the contrary, the peaks are suppressed and broadened; see Figure 6b. Therefore we can acquire the better defined phase of the cavity field by adjusting the detunings.

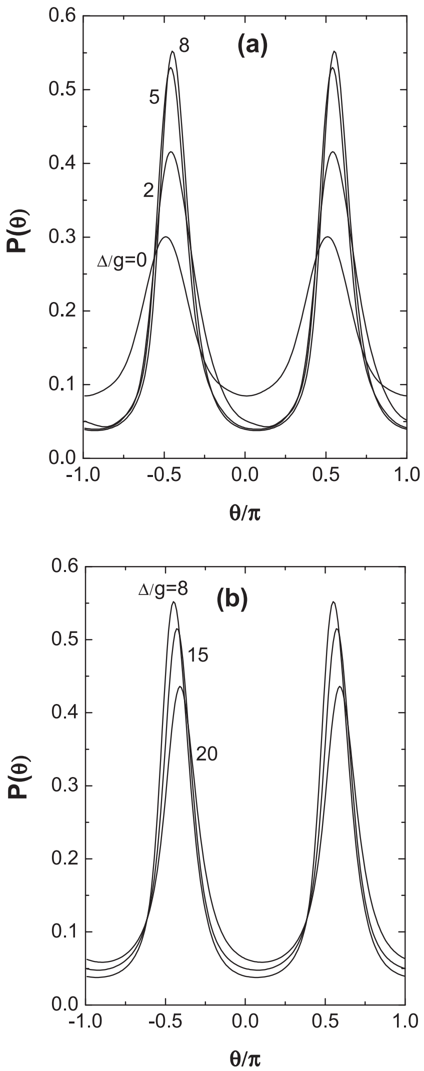

Fig. 6. The PPD of the field in the cavity pumped by the coherent atoms with different detunings. Atoms are injected in superposition states with $|C_a/C_c| = 0.88$, $|C_b| = 0$ and $\phi = 0$, and $g\tau = 0.482\pi$.

3.3 The case when the atom is in the superposition of the upper, intermediate and lower states

As shown above, the one-photon transition process can induce a single peak and the two-photon transition process can induce double peaks in the phase distribution. It seems we can discern which transition process happens in the atom-field interaction by resorting to the PPD of the cavity field. If the cavity is pumped by the atoms in the superposition of the upper, intermediate and lower state, which can be expressed as follows: $C_a = 1/2e^{i\pi}$, $C_b = \sqrt{2}/2e^{i\phi}$, and $C_c = 1/2e^{i\pi/3}$, the one- and two-photon transition process will coexist, and a competing phenomenon will appear in the PPD of the cavity field. Without general loss, we choose the phase in C_a and C_c as π and $\pi/3$, respectively.

From Figure 7, we can observe the most peaked $P(\theta)$ at $\theta = 5\pi/6$, if we adjust the phase of the intermediate state as $\phi = (\pi + \pi/3)/2 = 2\pi/3$. In this case the one-photon transition process dominates the emission of the injected atoms. The peak becomes broader if ϕ shifts from $\phi = 2\pi/3$ to $\pi/6$ gradually, and the double peaks are acquired at $\phi = \pi/6$ which shows the two-photon transition process taking place. However, if ϕ is adjusted from $\pi/6$ to $-\pi/3$, the peak at $\theta = 5\pi/6$ disappears, while the peak at $\theta = -\pi/6$ becomes narrower and more remarkable. When $\phi = -\pi/3$, finally, the most remarkable peak appears at $\theta = -\pi/6$, which shows the best-defined phase of the field in the cavity.

According to Figure 7, the double-peak phase distribution is acquired at $\phi = \pi/6$, and the height of the peaks can be adjusted if we modulate the population amplitudes of states $|a\rangle$, $|b\rangle$ and $|c\rangle$. As shown in Figure 8, if we let $|C_a| = |C_c|$, the phase distribution becomes more peaked

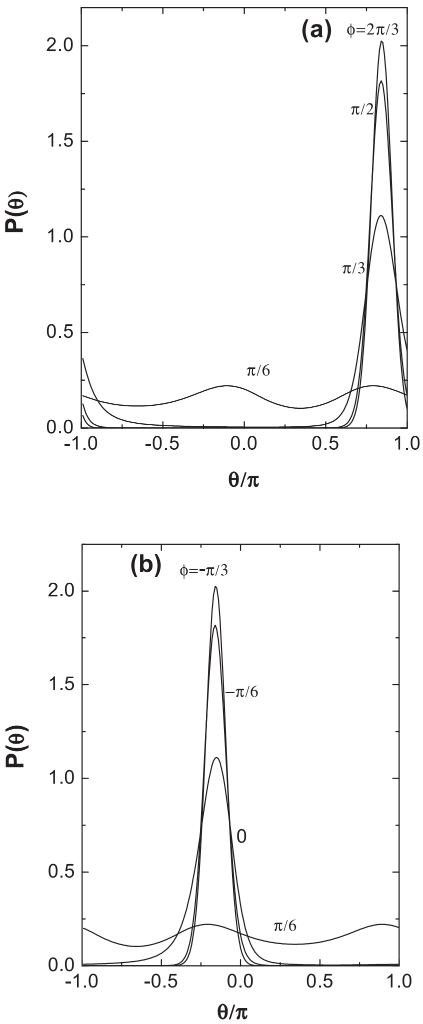

Fig. 7. The PPD of the field in the cavity pumped by the coherent atoms with different phases of the intermediate state. The atoms are injected in the superposition state with $C_a = 1/2e^{i\pi}, C_b = \sqrt{2}/2e^{i\phi}, C_c = 1/2e^{i\pi/3}$ and the phase ϕ of the intermediate state is adjusted as labeled.

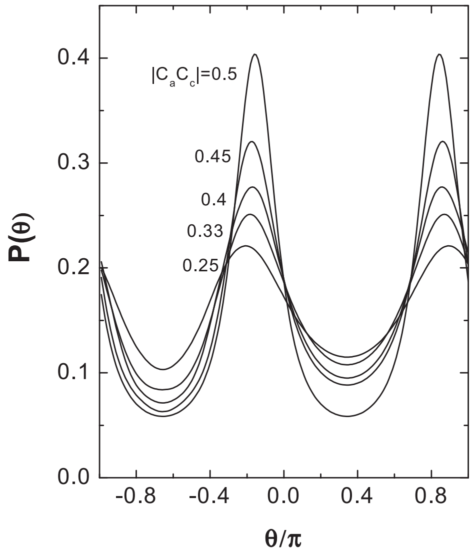

Fig. 8. The PPD of the field in the cavity pumped by the coherent atoms with different amplitudes. The atoms are injected in such a superposition state with $\phi = \pi/6$, $|C_a| = |C_c|$, $C_a = |C_a|\, e^{i\pi}$, $C_b = |C_b|\, e^{i\phi}$, $C_c = |C_c|\, e^{i\pi/3}$ and $|C_aC_c| = 0.25, 0.33, 0.4, 0.45, 0.5$.

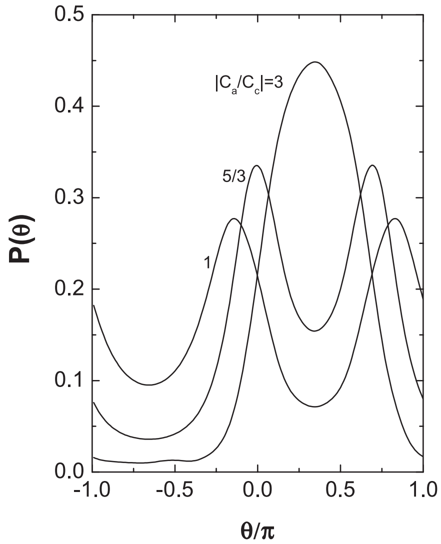

Fig. 9. The phase distribution of the field in the cavity pumped by the coherent atoms with different amplitudes. The atoms are injected in such a superposition state with $\phi = \pi/6$, $|C_b| = 0.2$, $C_a = |C_a|\, e^{i\pi}$, $C_b = |C_b|\, e^{i\phi}, C_c = |C_c|\, e^{i\pi/3}$ and $|C_a/C_c| = 1, 5/3, 3$.

with increasing the value of $|C_aC_c|$. However if $|C_a| \neq |C_c|$, the phase distribution displays the competition between the one-photon and two-photon transition process, as seen from Figure 9. When we increase the value of $|C_a/C_c|$ and leave $C_b = 0.2$ unchanged, the double peaks will turn to one peak gradually. Thus the two-photon transition process will be replaced by the one-photon transition process slowly as increasing the value of $|C_a/C_c|$.

We employ the semi-classical theory [18] to explain the procedure of phase competition when $|C_b^* C_a| = |C_c^* C_b|$. If the atom transits from $|a\rangle$ to $|c\rangle$ directly, it will emit two photons at the same time, which will induce double peaks in the phase distribution. If the atom transits from $|a\rangle$ to $|b\rangle$ or $|b\rangle$ to $|c\rangle$, it will emit one photon with the phase $\phi = \phi_a - \phi_b$ or $\phi = \phi_b - \phi_c$, which will excite a single peak in the phase distribution. The one-photon transition process can be understood as the radiation of an electric dipole. As to an atom in the cascade three-level configuration, the dipole can be written as:

$$P(t) = |C_b^* C_a|\, \wp_{ba} \exp\left(i\left(\phi_a - \phi_b\right) - i\left(\omega_a - \omega_b\right) t\right) + |C_c^* C_b|\, \wp_{cb} \exp\left(i\left(\phi_b - \phi_c\right) - i\left(\omega_b - \omega_c\right) t\right) + \text{c.c.} \quad (15)$$

where

$$\wp_{ba} = e\langle b|r|a\rangle, \quad \wp_{cb} = e\langle c|r|b\rangle.$$

In our system, we get

$$\wp_{ba} = \wp_{cb}, \quad \omega_a - \omega_b = \omega_b - \omega_c = \omega. \quad (16)$$

If $|C_b^* C_a| = |C_c^* C_b|$ and we let $|C_b^* C_a| \wp_{ab} = |C_c^* C_b| \wp_{bc} = A$, the dipole can be expressed as:

$$P(t) = A [\exp(i(\phi_a - \phi_b) - i\omega t) + \exp(i(\phi_b - \phi_c) - i\omega t)] + \text{c.c.} \quad (17)$$

From a semi-classical perspective, then the field radiated is a sum of two terms, which will interfere together in the cavity:

$$E^{(+)} e^{i\omega t} = \xi \exp(i(\phi_a - \phi_b)) + \xi \exp(i(\phi_b - \phi_c)), = 2\xi \cos\left(\frac{\phi_a + \phi_c}{2} - \phi_b\right) \exp\left(i \frac{\phi_a - \phi_c}{2}\right). \quad (18)$$

According to equation (18), the photon intensity will reach a maximum at $\phi_b = (\phi_a + \phi_c)/2$, $(\phi_a + \phi_c)/2 - \pi$, and zero at $\phi_b = (\phi_a + \phi_c)/2 \pm \pi/2$. Usually the peak of the phase distribution is positively relevant to the photon intensity, so the phase distribution will become more peaked with increasing the photon intensity. When the photon intensity becomes zero, the single peak induced by the one-photon process will disappear and the phase of the field will be defined by the two-photon process only.

As shown in Figure 2, the peak of the phase distribution appears at $\theta = \pi/2$ when $\phi_a - \phi_b = 0$, then we set $\pi/2$ as a reference point. As we change the difference between$\phi_a$and ϕ_c, the peak will move according to equation (18). It is very easy to decide the place where the peak appears from equation (18), that is if $\phi_b \in [(\phi_a + \phi_c)/2 - \pi/2, (\phi_a + \phi_c)/2 + \pi/2]$, the peak will appear at $\pi/2 + (\phi_a - \phi_c)/2$, or else if $\phi_b \in [(\phi_a + \phi_c)/2 + \pi/2, (\phi_a + \phi_c)/2 + 3\pi/2]$, the peak will appear at $-\pi/2 + (\phi_a - \phi_c)/2$.

4 Experimental consideration

We should point out that the parameters in equation (14) are without loss of generality. For different interaction times, one can obtain similar PPDs, as shown in Figures 5 and 6. In this sense our results are robust to an experimental observation. When $g\tau = 0.13\pi$, the cavity field is very strong, so that the PPDs become more pronounced. To observe the predicted results in the present paper based on equation (13), an experimentalist must obtain density matrix elements of the cavity mode in the steady state. Coupling an electromagnetic field out of a "black-box" micromaser cavity is out of the question, because this operation would drastically change the field state in the cavity. Information on the cavity field must be obtained from probe atoms. Several nondestructive schemes are contributed to measure quantum states inside a cavity [19–22]. Very recently, in reference [22] the authors have directly measured the complete Wigner function W of the vacuum and of a single-photon state for a field stored in a high- Q cavity. Extensions to other cavity field states are also within reach. One can extract the corresponding field-state density matrix elements from the measured Wigner function by resorting to the procedure described in reference [23]. Note that in reference [23] the researchers have experimentally reconstructed the density matrices and Wigner functions for various quantum states of motion of a $^9Be^+$ ion harmonically bound in a trap.

5 Conclusions

In this paper we have formulated the master equation of the cavity field pumped by atoms in the superposition of the upper, intermediate and lower state and simulated the quantum dynamics of the cavity field by the Monte Carlo wave function approach. We have investigated the phase distribution of the cavity field when the pumping atom is initially prepared in three different superposition states and focused on the transfer of atomic coherences to the cavity field by adjusting the relative phases and the amplitudes of the polarized atoms, and the detunings between the atom and cavity.

The phase distribution turns out to be a single peak when the cavity is pumped by the atoms in the superposition of the upper and intermediate state or the intermediate and lower state. The preferred phase of the cavity field is shifted according to the relative phases of the coherent atoms and the atom-field detunings. If the cavity is pumped by the atoms in the superposition of the upper and the lower states, double peaks take place in the phase distribution of the cavity field. The double peaks are shifted according to half relative phases of the coherent atoms. And if we add appropriate detunings, the double peaks become narrower and more remarkable. When we take the atoms in the superposition of the upper, intermediate and lower states as pumping resources, the phase distribution of the cavity-field becomes very complicated. The phase distribution displays the competition between a single peak and double peaks which reflects the one- and two-photon transition process, respectively. The amplitudes and phases of the coherent atoms play a very important role in the phase distribution of the cavity field.

Our investigation shows that it is perfect to manipulate the phases of the field in a micromaser by adjusting the relative phases and the amplitudes of the polarized atoms, and the detunings between the atom and cavity.

Finally the state of the art of one-atom experiments in the microwave regime allows for the experimental observation of the predicted phenomena.

This work is in part supported by the National Natural Science Foundation of China. X.S. Chen would like to thank Chinese National Key Basic Research Special Fund, Key Fund of Chinese National Science Foundation (10234040), Key Fund of Shanghai Science and Technology Foundation (02DJ14066).

References

1. *The Physics of Quantum Information*, edited by D. Bouwmeester, A. Ekert, A. Zeilinger (Springer, Berlin, 2000)
2. E.T. Jaynes, F.W. Cummings, Proc. IEEE **51**, 89 (1963)

3. D. Meschede, H. Walther, G. Müller, Phys. Rev. Lett. **54**, 551 (1985)
4. M. Weidinger, B.T.H. Varcoe, R. Heerlein, H. Walther, Phys. Rev. Lett. **82**, 3795 (1999)
5. B.T.H. Varcoe, S. Brattke, M. Weidinger, H. Walther, Nature **403**, 743 (2000)
6. F. Casagrande, A. Lulli, J. Opt. B: Quant. Semiclass. Opt. **4**, S260 (2002)
7. F. Casagrande, A. Lulli, V. Santagostino, Phys. Rev. A **65**, 023809 (2002)
8. S.D. Du, Y. Tanimura, J. Opt. B **4**, 402 (2002), and references therein
9. M. Brune, J.M. Raimond, S. Haroche, Phys. Rev. A **35**, 154 (1987)
10. L. Davidovich, J.M. Raimond, M. Brune, S. Haroche, Phys. Rev. A **36**, 3771 (1987)
11. M. Brune, J.M. Raimond, P. Goy, L. Davidovich, S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987)
12. Imrana Ashraf, J. Gea-Banacloche, M.S. Zubairy, Phys. Rev. A **42**, 6704 (1990)
13. A.H. Toor, S. Y. Zhu, M.S. Zubairy, Phys. Rev. A **53**, 3529 (1996)
14. J. Dalibard, Y. Castin, K. Mølmer, Phys. Rev. Lett. **68**, 580 (1992)
15. E.S. Guerra, C.R. Carvalho, J. Opt. B: Quant. Semiclass. Opt. **3** 260 (2001)
16. G. Lindblad, Commun. Math. Phys. **48**, 119 (1976)
17. S. M. Barnett, D.T. Pegg, *Quantum Fluctuations*, edited by S. Reynaud, E. Giacobino, J. Zinn-Justin (Elsevier, Amsterdam 1997), pp. 564-575
18. *Quantum Optics*, edited by M.O. Scully, M.S. Zubairy (Cambridge University Press, 1997)
19. P.J. Bardroff et al., Phys. Rev. A **51**, 4963 (1995); P.J. Bardroff et al., Phys. Rev. A **53**, 2736 (1996)
20. Si-de Du, Lu-wei Zhou, Zhi-zhan Xu, Wen-qi Zhang, Yuan Li, J. Phys. B: At. Mol. Opt. Phys. **31**, 4895 (1998)
21. L.G. Lutterbach, L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997)
22. P. Bertet et al., Phys. Rev. Lett. **89**, 200402 (2002)
23. D. Leibfried et al., Phys. Rev. Lett. **77**, 4281 (1996)